

Solution to Problem A (Willem-Alexander spins a coin ...)

Steps of MLE
computation
Applied rules

a) $X = \{T, T, T, T, T, T, H\}$

θ is the unknown parameter representing the probability that the coin comes up heads

We are asked to compute the likelihood of θ , $p(X; \theta)$.

step 1: $p(X; \theta) = \underbrace{(1-\theta) \cdot (1-\theta) \dots (1-\theta)}_{\substack{\text{i.i.d.} \\ \text{sample} \\ 6 \text{ times}}} \theta = (1-\theta)^6 \theta$

b) step 2: $\log p(X; \theta) = \log ((1-\theta)^6 \theta) = \log ((1-\theta)^6) + \log \theta$
 $+ \log \theta = 6 \log (1-\theta) + \log \theta$
product rule
power rule

step 3: $\frac{\partial}{\partial \theta} \log p(X; \theta) = \frac{\partial}{\partial \theta} [6 \log (1-\theta) + \log \theta] =$
 $= -\frac{6}{1-\theta} + \frac{1}{\theta}$
derivative of a log & chain rule

step 4: $-\frac{6}{1-\theta} + \frac{1}{\theta} = 0$
 $\frac{1}{\theta} = \frac{6}{1-\theta}$
 $1-\theta = 6\theta$
 $\hat{\theta} = \frac{1}{7}$