Discussion 2

18 Sept 23

Announcements

- Homework 1 due this Wednesday at 10.10am
- OH from 5 to 7pm today at CCDS B64

Problem 1

A bag contains 4 red balls and 6 blue balls. A ball is taken at random from the bag and not put back in. A second ball is removed from the bag. What is the probability that both balls are of different colours?

Solution

$$P(RB) = \frac{4}{10} \cdot \frac{6}{9} = \frac{24}{90} = \frac{12}{45}$$

$$P(BR) = \frac{6}{10} \cdot \frac{4}{9} = \frac{24}{90} = \frac{12}{45}$$

$$P(\text{both the balls are of different colors}) = \frac{12}{45} + \frac{12}{45} = \boxed{\frac{8}{15}}$$

Problem 2

Sage can either take a course in CV or in NLP.

If Sage takes the course in CV, then he will receive an A grade with probability 1/2; if he takes the course in NLP then he will receive an A grade with probability 1/3.

Sage decides to base his decision on the flip of a fair coin. What is the probability that Sage will get an A in NLP?

Solution

Let N be the event that Sage takes NLP.

Let A be the event of receiving the desired grade (A).

The conditional probability of Sage receiving an A in NLP given that he selects the course $P(A|N) = \frac{1}{3}$

The probability of selecting NLP $P(N) = \frac{1}{2}$

Hence, probability of $P(AN) = P(N)P(A|N) = \frac{1}{3} . \frac{1}{2} = \boxed{\frac{1}{6}}$

Review of useful distributions

Geometric Distribution

- Tracks how many attempts (failures) it takes to achieve a desired outcome (success)
- The number of times you need to flip a coin until you get heads

Binomial Distribution

- Predicts the likelihood of a specific outcome (success or failure) in situations with only two possible results
- The probability of making a free throw in basketball

Exponential Distribution

- Estimates the time you have to wait until something happens
- The time between arrivals of buses at a bus stop

Review of useful distributions

Poisson Distribution

- Represents the number of rare events occurring in a fixed interval of time or space
- The number of customer arrivals at a store in an hour.

Uniform Distribution

- Ensures that every outcome in a range is equally likely
- Rolling a fair six-sided die, where each number has a 1/6 chance of occurring.

Gaussian Distribution (Normal Distribution)

- Describes data where most values cluster around the mean
- The heights of people in a population, forming a bell-shaped curve.

Problem 3

Find $E[X]$ where X is the outcome when we roll a fair die.

Solution

Since $p(1) = p(2) = p(3) = p(4) = p(5) = p(6) = 1/6$, we obtain $E[X] = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6) = 7/2$

Problem 4

One archer has an 80% probability of hitting a target, while another has only a 70% probability of hitting a target. What is the probability of the target being hit (at least once) if both archers fire at it simultaneously?

Solution

We assume that the events are independent. Then the probability that both archers miss is P(S1 misses,S2 misses) = $0.2 \cdot 0.3$ and the probability of at least one hit is P(at least one hit) = $1 - (0.2 \cdot 0.3) = \boxed{0.94}$.

Problem 5

Prove that if $P(A|B)>P(A)$, then $P(B|A)>P(B)$. You can assume that all probabilities are nonzero.

Solution

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

So we have

$$\frac{P(A,B)}{P(B)} > P(A).$$

Dividing through by $P(A)$ and multiplying through by $P(B)$ we get,

$$\frac{P(A,B)}{P(A)} > P(B)$$

So,

$$\boxed{P(B \mid A) > P(B)}$$