Discussion 3

25 Sept 23

Announcements

- Homework 2 due on Wednesday at 10.10am
- OH at CCDS B64 from 5 to 7pm
- Best way to reach out is through Piazza (private post)

Problem 1

Consider discrete random variables X and Y with the following probability mass function (pmf):

x	y	$P(X = x, Y = y)$
1	1	0.1
2	2	0.2
3	3	0.3
4	4	0.4

Find $E[X^2Y]$.

Solution

The expected value (or mean) of a discrete random variable is given by:

$$E[X^2Y] = \sum_x \sum_y (x^2y)P(X = x, Y = y)$$

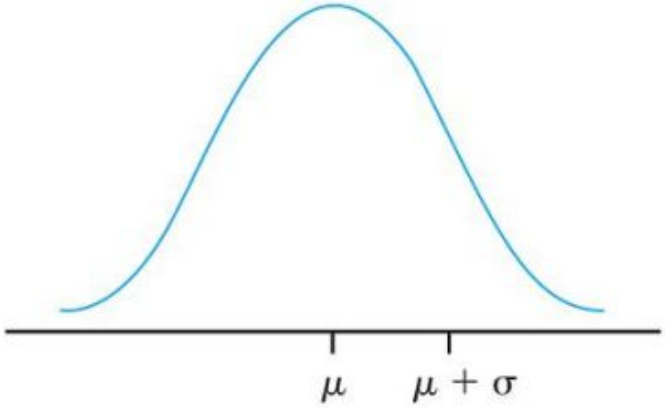
$$\begin{aligned} E[X^2Y] &= (1^2 \cdot 1 \cdot 0.1) + (2^2 \cdot 2 \cdot 0.2) + (3^2 \cdot 3 \cdot 0.3) + (4^2 \cdot 4 \cdot 0.4) \\ &= (1 \cdot 1 \cdot 0.1) + (4 \cdot 2 \cdot 0.2) + (9 \cdot 3 \cdot 0.3) + (16 \cdot 4 \cdot 0.4) \\ &= 0.1 + 1.6 + 8.1 + 25.6 \\ &= \boxed{35.4} \end{aligned}$$

Normal distribution

A continuous r.v. *X* is said to have a **normal distribution** with parameters μ and $\sigma > 0$ (or μ and σ^2), if the pdf of *X* is

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \text{ where } -\infty < x < \infty$$

Normal distribution

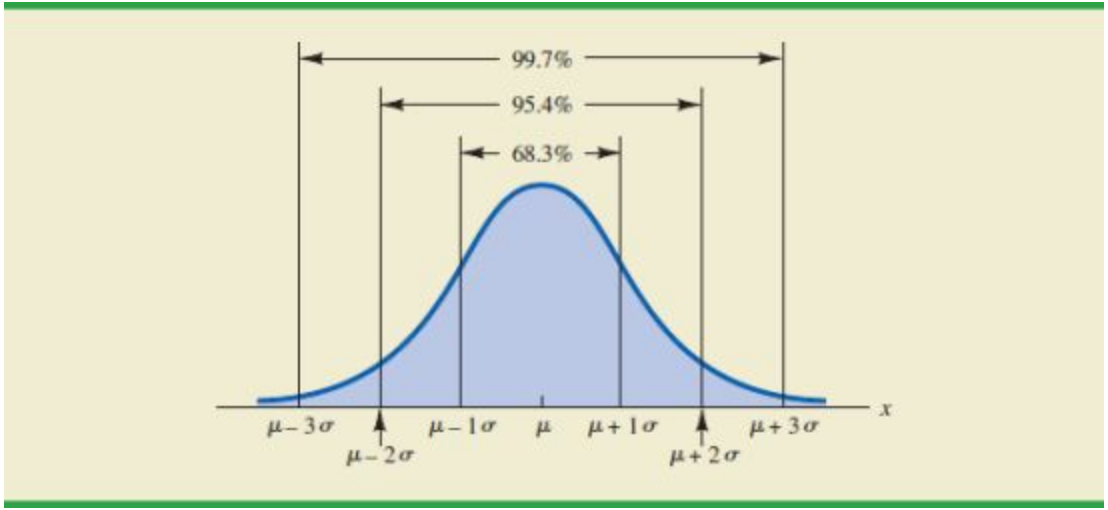


Visualizing μ and σ for a normal distribution

Problem 2

A normal distribution has a mean of 100 and a standard deviation of 10. What is the probability that a randomly selected value falls between 90 and 110?

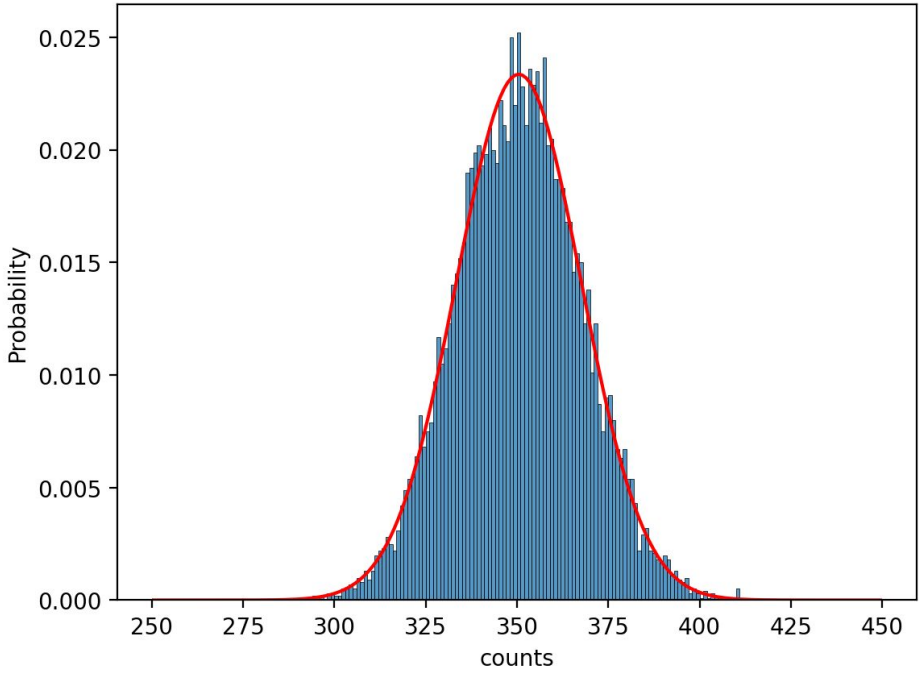
Practice problem



Using this, p = 0.683

Central limit theorem

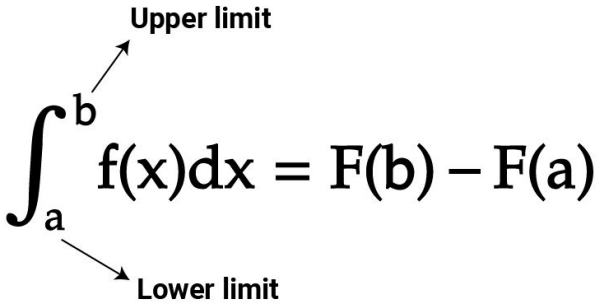
No matter the distribution of data, the means of the samples are normally distributed



Resources:

1. https://www.youtube.com/watch?v=YAIJCEDH2uY - intuitive idea

A note on Integrals



$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$n \neq -1$$

Problem 3

Consider a random variable X. Its PDF is given by:

$$f(x) = \begin{cases} x^6, 3 < x < 7 \\ 0, \text{otherwise} \end{cases}$$

Find $E[X^2]$.

Solution

To find $E[X^2]$, we use LOTUS.

$$E[X^2] = \int_3^7 x^2 . x^6 dx$$

$$E[X^2] = \int_3^7 x^8 dx$$

$$= \left[\frac{x^9}{9}\right]_3^1$$

$$= \frac{7^9}{9} - \frac{3^9}{9}$$

Note - cheat sheet for integrals

Problem 4

Find $E[X^2]$ given the PDF:

$$f(x) = \begin{cases} x+1, 3 < x < 7 \\ 0, otherwise \end{cases}$$

Solution

$$\int x^2(x+1)dx = \int x^3 + x^2 dx$$

with limits from 3 to 7. So, we have:

$$\frac{x^4}{4} + \frac{x^3}{3}$$

Putting the upper limit and then subtracting the lower limit from it,

$$\frac{7^4}{4} + \frac{7^3}{3} - \frac{3^4}{4} - \frac{3^3}{3}$$