

# DS 122 Practice Final Exam

Name:

BUID:

## Problem A (3 points)

A company ships packages that are variable in weight, with an average weight of 15 lbs and a standard deviation of 10. Assuming that the packages come from a large number of customers so that it is reasonable to model their weights as independent random variables, find the probability distribution of the total weight of 100 packages.

## Problem B (6 points)

You are running a (very small) experiment on coin flipping comparing a "control" coin to a "treated" coin. You flip each coin once and observe the first coin comes up heads and the second coin comes up tails. Let's call the probability the "control" coin comes up heads  $p_1$  and the probability the treated coin comes up heads  $p_2$ .

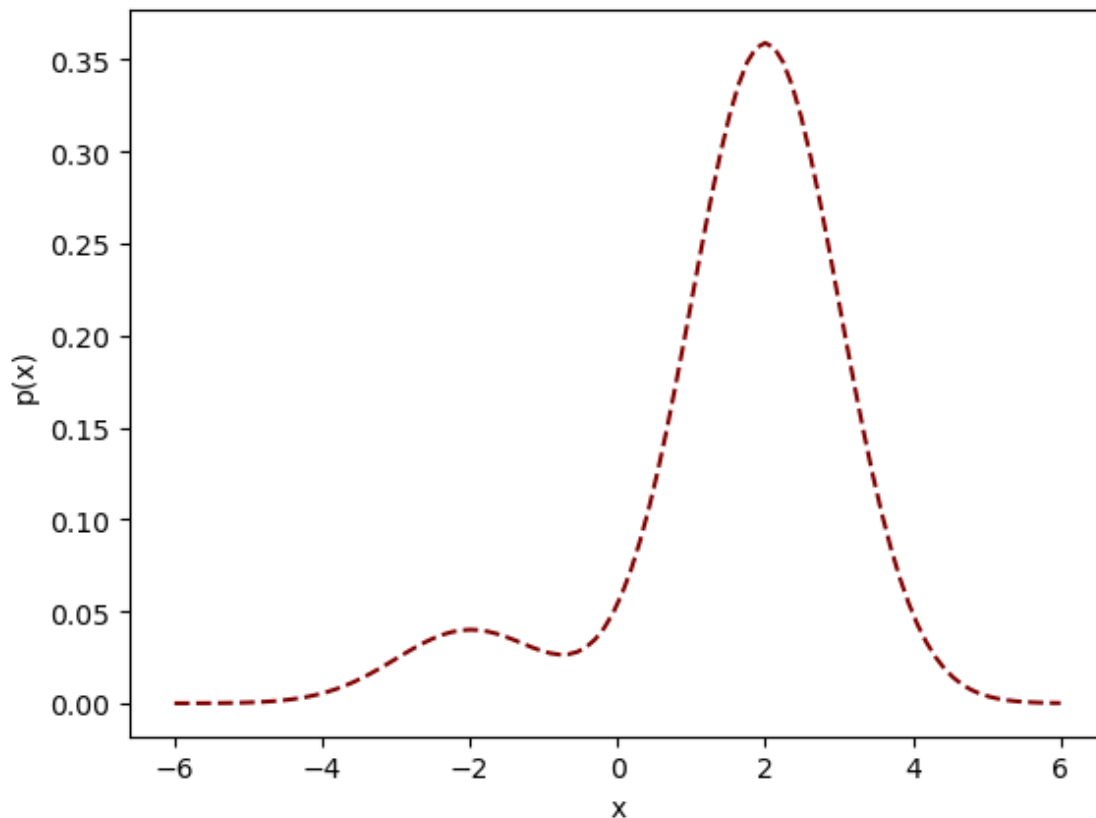
You want to estimate the parameters  $p_1$  and  $p_2$  but you are short on time so you only consider the possibility that each coin is either fair ( $p = 0.5$ ) rigged for tails ( $p = 0.1$ ) or rigged for heads ( $p = 0.9$ ). Starting with a uniform prior, what are the posterior probabilities for the two parameters for the three possible hypotheses?

## Problem C (3 points)

Last season the Boston Bruins scored 305 goals over the course of 82 games. So far this season the Bruins have scored 87 goals in 26 games (note: we treat all games as independent events). Using last years scoring rate as a prior modeled by a Gamma distribution, what is the posterior distribution of the the goal scoring rate for this season and what is the expected value of the distribution (hint: the expected value of a Gamma distribution is  $\alpha/\beta$ )?

## Problem D (3 points)

You are using the Metropolis-Hastings algorithm to sample from the following complex distribution:



For simplicity, assume that in the plot above, the probability  $p(x)$  at  $x=0$  is 0.05 and at  $x=3$  is 0.3.

Assume candidates are selected using a normal distribution around the current state with a standard deviation of 1.

Consider a transition from 3 to 0. What is the probability this transition will be accepted?

### Problem E (7 points)

Let  $x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)}$  be a sample from a distribution with probability density

$$p_{\theta}(x) = \theta x^{\theta-1} \text{ for } x \in (0, 1)$$

and 0 otherwise. Here  $\theta > 0$  is the unknown parameter. Determine the maximum likelihood estimator for  $\theta$ .

### Problem F (12 points)

With a primary diet of leaves, which are not very nutritional and are hard to digest, red howler monkeys spend most of their time eating and resting. Eating (E) and resting (R) are considered to be their primary states. Suppose that scientists observe that, on average, the monkeys spend 30% of their time eating and 70% resting. Suppose also that the biologists determined

that if a monkey is eating at hour  $n$ , then there is a 60% probability that the animal will be eating the next hour,  $n + 1$ . If a monkey is resting at hour  $n$ , then the next hour,  $n + 1$ , it will remain resting with probability 80%.

Moreover, suppose that a group of scientists wants to study the behaviour of a red howler monkey in a more remote area. Knowing they will have limited opportunities of making visual observations, they attach a small microphone to one of the monkeys, whom they name Holly. The biologists discern that when hearing munching (M), the monkey is probably eating; but when hearing breathing (B) noises, the animal is likely to be at rest. The scientists analyze the noises once an hour. However, the microphone results are not always accurate. Besides background noises, such as from rain or other monkeys, a sleeping Holly might be moving her mouth. The scientists assume that there is 90% chance that if Holly is resting, then their system indicates breathing noises. They also assume that their system is only 80% accurate in detecting munching noises, while Holly is eating.

The first recording received by the scientists has been interpreted as munching. The recording that was received an hour later has been interpreted as breathing.

### Part 1 (3 points)

Provide the initial state vector  $\pi$ , transition matrix  $A$ , and emission matrix  $B$  for the described problem.

### Part 2 (7 points)

Perform the initialization and forward pass of the Viterbi algorithm.

### Part 3 (2 points)

Find which primary states correspond to the first two recordings received by the scientists by performing the backward pass.

## Problem G (4 points)

You would like to find a slope  $\beta_1$  and intercept  $\beta_0$  for a line that minimizes the sum of squared error between predicted and observed values. Because your dataset has billions of datapoints, you decide to use stochastic gradient descent.

If you initialize the slope to -4 and the intercept to 0, set the learning rate to 0.1 and the first random data point you select is (2,5), what will be the new slope and intercept after one step of stochastic gradient descent?