# DS 122 Homework 5

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# Contents

# 1 Q1 Problem A

Understanding combined events: A fair dice is rolled twice.

## 1.1 Q1.1

What is the probability that both rolls result in a 6?

### 1.1.1 Solution

To determine the probability that both rolls result in a 6:

1. The probability that the first roll is a 6 is $P(6) = \frac{1}{6}$
2. The probability that the second roll is a 6 is also $P(6) = \frac{1}{6}$

Since the rolls are independent events, you multiply the probabilities:

$$P(\text{both 6's}) = P(6) \cdot P(6) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

So, the probability that both rolls result in a 6 is

$$\frac{1}{36}$$

.

## 1.2 Q1.2

If the first roll is a 5, compute the probability that the second roll is also a 5.

### 1.2.1 Solution

The outcome of the second roll is independent of the outcome of the first roll. Therefore, the probability that the second roll is a 5 remains unchanged regardless of the result of the first roll.

$$P(\text{second roll is 5} \mid \text{first roll is 5}) = P(\text{second roll is 5})$$

The probability that the second roll is a 5 is $P(5) = \frac{1}{6}$

So, given that the first roll was a 5, the probability that the second roll is also a 5 is

$$\frac{1}{6}$$

### 1.3 Q2 Problem B

Assume that the probability of a person having a common cold in winter is 30%. Also, suppose the probability of someone sneezing in winter is 50%. Given that the probability of someone with a cold sneezing is 80%, if a person sneezes in winter, what is the probability that they have a common cold?

### 1.4 Solution

Given:

- $P(\text{Cold})$ = Probability of having a cold = 0.30
- $P(\text{Sneeze})$ = Probability of sneezing in winter = 0.50
- $P(\text{Sneeze}|\text{Cold})$ = Probability of sneezing given that someone has a cold = 0.80

Using Bayes' theorem:

$$P(\text{Cold}|\text{Sneeze}) = \frac{P(\text{Sneeze}|\text{Cold}) \cdot P(\text{Cold})}{P(\text{Sneeze})}$$

Substituting in the values:

$$P(\text{Cold}|\text{Sneeze}) = \frac{0.8 \cdot 0.3}{0.5} = 0.48$$

So, if a person sneezes in winter, the probability that they have a common cold is

$$0.48$$

or 48%.

## 2 Q3 Problem C

Imagine you're in a game show. In front of you is a box containing three dice: a 6-sided die, an 8-sided die, and a 12-sided die. The host explains that one die will be chosen at random, and if you can correctly identify which die was chosen based solely on the roll outcome, you'll win a prize.

The host picks a die at random, rolls it, and announces: "The outcome is 7!"

Given this information, use a Bayes table to find what the probability is that the host chose the 8-sided die? You must write out the full Bayes table for full credit!

### 2.1 Solution

Let's set up a Bayes table:

| **Event** | **Prior $P(D_i)$** | **Likelihood $P(\mathbf{R7}|\mathbf{Di})$** | **Joint Probability** | **Marginal Likelihood** |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| $D_6$ | $\frac{1}{3}$ | 0 | 0 | ? |
| $D_8$ | $\frac{1}{3}$ | 1/8 | 1/24 | ? |
| $D_{12}$ | $\frac{1}{3}$ | 1/12 | 1/36 | ? |

The marginal likelihood, $P(\text{R7})$, is the sum of the joint probabilities, which is

$$\frac{1}{24} + \frac{1}{36} = \frac{5}{72}$$

Using Bayes' theorem:

$$P(\text{D8} \mid \text{R7}) = \frac{P(\text{R7} \mid \text{D8}) \cdot P(\text{D8})}{P(\text{R7})}$$

$$P(\text{D8} \mid \text{R7}) = \frac{\frac{1}{8} \cdot \frac{1}{3}}{\frac{5}{72}} = \frac{9}{40}$$

So, given that the outcome is 7, the probability that the host chose the 8-sided die is $\frac{9}{40}$.

# 3 Q4 Computational

Please download the file from Resources section under Piazza or from the following link https://piazza.com/class_profile/get_resource/llqwp5rdfue104/lnwktz66xts5sb