DS 122 Homework 9

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1 Problem 1

It's not possible to directly observe the mood of a person. However, facial expressions are observable and can be used to predict someone's mood. For simplicity, we assume that there are two categories of mood: good and bad. We also assume that there are three possible facial expressions: smiling, neutral, and frowning. When we first meet a person, there is a 70% chance that the person is in a good mood and a 30% chance that the person is in a bad mood.

The transition and emission probabilities can be found in the state-transition diagram below.

Assume that the mood changes on a daily basis

1.1 Q1.1

Find the initial probabilities, transition matrix, and emission matrix.

1.2 Solution

Initial Probability Vector:

$$\pi = \begin{Bmatrix} 0.7 \\ 0.3 \end{Bmatrix}$$

Transition Matrix with column sums to 1:

$$A = \begin{Bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{Bmatrix}$$

Emission Matrix with column sums to 1:

$$B = \begin{Bmatrix} 0.5 & 0.2 \\ 0.4 & 0.35 \\ 0.1 & 0.45 \end{Bmatrix}$$

1.3 Q1.2

Perform the initialization stage of the Viterbi algorithm. Show all your steps.

1.4 Solution

Given:

Initial Probabilities
- Probability of Good Mood initially: 0.7
- Probability of Bad Mood initially: 0.3

Emission Probabilities for "Frowning"
- Probability of Frowning given Good Mood: 0.1
- Probability of Frowning given Bad Mood: 0.45

Multiply the initial state probabilities with the emission probabilities for the first observation.

For 'Good' Mood:

$$\text{Viterbi (Good)} = P(\text{Good}) \cdot B(\text{Frowning} \mid \text{Good}) = 0.7 \times 0.1 = 0.07$$

For 'Bad' Mood:

$$\text{Viterbi (Bad)} = P(\text{Bad}) \cdot B(\text{Frowning} \mid \text{Bad}) = 0.3 \times 0.45 = 0.135$$

These values represent the maximum probability of being in each state given the first observation.

1.5 Q1.3

Perform the forward pass of the Viterbi algorithm. Show all your steps.

1.6 Solution

For "Smiling":

$$\text{Viterbi(Good)} = \max[\text{Viterbi(previous state)} \cdot \text{Transition to Good}] \cdot \text{Emission(Smiling|Good)}$$

$$\text{Viterbi(Bad)} = \max[\text{Viterbi(previous state)} \cdot \text{Transition to Bad}] \cdot \text{Emission(Smiling|Bad)}$$

- From Good: $0.070 \cdot 0.8 = 0.056$
- From Bad: $0.135 \cdot 0.4 = 0.054$
- Max value: $\max(0.056, 0.054) = 0.056$
- Emission: $0.056 \times 0.5 = 0.028$

- From Good: $0.07 \cdot 0.2 = 0.014$
- From Bad: $0.135 \cdot 0.6 = 0.081$

- Max value: $\max(0.014, 0.081) = 0.081$
- Emission: $0.081 \cdot 0.2 = 0.0162$

Updated Viterbi Values after "Smiling":
- For Good Mood: 0.028
- For Bad Mood: 0.0162

For "Neutral":

$$\text{Viterbi(Good)} = \max[\text{Viterbi(previous state)} \cdot \text{Transition to Good}] \cdot \text{Emission(Neutral|Good)}$$

$$\text{Viterbi(Bad)} = \max[\text{Viterbi(previous state)} \cdot \text{Transition to Bad}] \cdot \text{Emission(Neutral|Bad)}$$

- From Good: $0.028 \cdot 0.8 = 0.0224$
- From Bad: $0.0162 \cdot 0.4 = 0.00648$
- Max value: $\max(0.0224, 0.00648) = 0.0224$
- Emission: $0.0224 \cdot 0.4 = 0.00896$

- From Good: $0.028 \cdot 0.2 = 0.0056$
- From Bad: $0.0162 \cdot 0.6 = 0.00972$
- Max value: $\max(0.0056, 0.00972) = 0.00972$
- Emission: $0.00972 \cdot 0.35 = 0.003402$

Updated Viterbi Values after "Neutral":
- For Good Mood: 0.00896
- For Bad Mood: 0.003402

1.7 Q1.4

Perform the backward pass of the Viterbi algorithm. Show all your steps.

1.8 Solution

Final State (after "Neutral"):

- Compare the Viterbi values for Good and Bad moods.
- Good Mood: 0.00896, Bad Mood: 0.003402

- The higher value is for Good Mood, so the most probable final state is Good Mood.

Second State (after "Smiling"):

- Determine which mood at the "Smiling" step was more likely to lead to a Good Mood at the "Neutral" step.
- Look at the transition probabilities from Good to Good and Bad to Good, and the Viterbi values at the "Smiling" step.
- Transition Probabilities: Good to Good (0.8), Bad to Good (0.4)
- Viterbi Values at "Smiling": Good Mood (0.028), Bad Mood (0.0162)
- The product of Viterbi value and transition probability is higher for Good Mood $(0.028 \cdot 0.8)$ than for Bad Mood $(0.0162 \cdot 0.4)$.
- Thus, the second state is also more likely to be Good Mood.

Initial State (after "Frowning"):

- Determine which mood at the "Frowning" step was more likely to lead to a Good Mood at the "Smiling" step.
- Transition Probabilities: Good to Good (0.8), Bad to Good (0.4)
- Viterbi Values at "Frowning": Good Mood (0.07), Bad Mood (0.135)
- The product of Viterbi value and transition probability is higher for Bad Mood $(0.135 \cdot 0.4)$ than for Good Mood $(0.07 \cdot 0.8)$.
- Thus, the initial state is more likely to be Bad Mood.

Most Probable Path:

- Initial State: Bad Mood (after observing "Frowning")
- Second State: Good Mood (after observing "Smiling")
- Final State: Good Mood (after observing "Neutral")

2 Problem 2 - Computational

- Add your answers in the same cell as the code or add another cell by copy pasting the existing cell
- Outputs from the answer key have been left as they are for your reference. My personal suggestion would be to create a new cell with the same code copied and make sure that the output coming is the same.
- Use of any LLM such as ChatGPT, Bing Chat are strictly forbidden unless otherwise specified in the questions

Link to Notebook

Please upload a copy of your completed computational notebook.