6.38

a. We identify the population mean

Property of the CLT: the expected value of the sample mean is equal to the population mean.

$\mu_{\bar{x}} = 3.65$

Standard deviation of the sample mean

$$\frac{0.15}{\sqrt{100}} = 0.015$$

b. Calculate the z-score.

$$3.67: \quad \frac{3.67 - 3.65}{\frac{0.15}{\sqrt{100}}} = 1.33$$

$$3.65: \quad \frac{3.65 - 3.65}{\frac{0.15}{\sqrt{100}}} = 0$$

We want to find $P(z \leq 1.33) - P(z \leq 0)$

z-score of $1.33 = 0.9074$
z-score of $0 = 0.5000$

$0.9074 - 0.5000 = 0.4074$

The probability that sample has a mean fuel cost between 3.65 and 3.67 is approximately 40.74%.

c. z-score of 3.67 is 1.33

$P(z > 1.33) = 1 - P(z \leq 1.33)$

Probability for z-score of $1.33 \approx 0.9074$

$1 - 0.9074 = 0.0926$

The probability that the sample has a mean fuel cost that exceeds 3.67 is approximately 9.26%.

d. If the sample size n is doubled from 100 to 200, the sampling distribution of $\bar{x}$ will become narrower.

- As n increases, the standard error decreases, which means the distribution of the sample mean becomes more concentrated around the population mean.

New SE with $n = 200$

$$\frac{0.15}{\sqrt{200}} \approx 0.0106$$

For part b: the z-score value would be smaller because the standard error is smaller, which means the probability would be smaller as well.

For part c: the z-score would be smaller because the standard error is smaller, which means the probability would be higher.

7.15

a. Point estimate for a population parameter is a single value of a statistic.

For the population mean, the best point estimate is the sample mean.

Sample mean = 19.3

Point estimate = $\bar{x}$ = 19.3

b. $\bar{x} \pm z \cdot \frac{s}{\sqrt{n}}$

Given: $\bar{x} = 19.3$, $s = 11.9$, and $n = 46$.

$$19.3 \pm \underbrace{1.96 \times \frac{11.9}{\sqrt{46}}}_{\text{Margin of error} \approx 3.43894}$$

$19.3 + 3.44 = 22.74$

$19.3 - 3.44 = 15.86$

95% confidence interval is approximately

(15.86, 22.74).

c. In practical terms, this means that we are 95% confident that the true average number of latex gloves used per week by all health care workers with a latex allergy lies between 15.86 and 22.74.

In other words, if we were to repeat this study many times, drawing different samples of 46 hospital employees each time, we would expect the average number of latex gloves used per week to fall between 15.86 and 22.74 in about 95% of the studies.

d. Randomness: The data should be a random sample from the population, ensuring that every individual in the population has an equal chance of being included in the sample.

Normality: · The population from which the sample is drawn is normally distributed.

· Sample size is large enough (usually $n \geq 30$) to invoke the Central Limit Theorem.

Independence: The observations in the sample should be independent of each other.

· 10% condition: If the sample size is less than 10% of the population size, we can treat the observations as being approximately independent.

7.25

a. Given that $\bar{x} = 1.13$, $s = 2.21$, and $n = 72$.

$$1.13 \pm 2.576 \times \frac{2.21}{\sqrt{72}}$$

$$(0.46, 1.80)$$

99% confident that the true average number of pecks made by chickens at blue string over a specified interval of time lie between 0.46 and 1.80.

In practical terms, if we were to repeat this experiment many times, choosing different samples of 72 chickens each time, we would expect the average number of pecks at the blue string to fall between 0.46 and 1.80 in about 99% of the experiments. This range provides an estimate of the average number of pecks at blue string among all chickens, taking into account the variability in pecking behavior among chickens.

b. Given that previous research has shown that the population mean number of pecks at white string is 7.5, this value is not within the confidence interval for the mean number of pecks at blue string.

This suggests that there is evidence that chickens are more apt to peck at white string than blue string. The average number of pecks at white string is significantly higher than the estimated range for the average for the average number of pecks at blue string.

However, this conclusion is based on the assumption that two populations are similar in all relevant aspects. If there are significant differences between them, for example, if different breeds of chickens were used in the two studies, then the conclusion may not be valid.

7.61

a. Population interest: The population of interest in this study is all American adults. This is the group that the study wants to make inferences about based on the sample data.

b. Sample for the study: The sample for the study is the 1000 American adults who were surveyed by Rasmussen Reports. This is the group from which data was actually collected and it's intended to be representative of the population of interest.

c. Parameter of Interest: The parameter of interest in this study is the proportion of all American adults who believe that Starbucks coffee is overpriced. This is the unknown value that the study is trying to estimate based on the sample data.

d. $\hat{p} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Given $\hat{p} = 0.73$, $z = 1.96$ (95% confidence interval), $n = 1000$

$0.73 \pm 1.96 \times \sqrt{\frac{0.73 \times 0.27}{1000}}$

$0.73 + 0.028 = 0.758$

$0.73 - 0.028 = 0.702$

The 95% confidence interval for the proportion of all American adult who believe that Starbucks coffee is overpriced lies between 70.2% and 75.8%.

7.84

a. The confidence coefficient is a measure of the level of confidence that we have in our estimate. It is typically expressed a decimal. For a 99% confidence interval, the confidence coefficient is 0.99.

b. The desired sampling error is the maximum difference that we would tolerate between the sample estimate and the true population parameter. In this case, the desired sampling error is 3 grams. This means that we want our estimate of the mean eye mass to be within 3 grams of the true mean eye mass.

c. $n = \left(\frac{z \cdot s}{E} \right)^2$

Given that $z = 2.576$, $s = 8.6$, and $E = 3$ grams.

$n = \left(\frac{2.576 \times 8.6}{3} \right)^2 \approx 55$

In order to estimate the mean eye mass of giraffes to within 3 grams with 99% confidence, we would need to measure the eye mass of about 55 giraffes.

7.88 We want to calculate the sample size needed to estimate a population proportion with a specified level of confidence and precision.

Formula: $n = \frac{Z^2 \cdot \hat{p} \cdot (1-\hat{p})}{E^2}$

In this case, we don't have an estimate for the population proportion ($\hat{p}$).

- Use the most conservative estimate, which is $\hat{p} = 0.5$.
  - It maximizes the product $\hat{p} \cdot (1-\hat{p})$, and therefore gives us the largest possible sample size, ensuring that our estimate will be precise enough.

Given that $Z = 1.645$, $\hat{p} = 0.5$, and $E = 0.02$.

$$n = \frac{(1.645)^2 \times 0.5 \times 0.5}{(0.02)^2} \approx 1692$$

In order to estimate the proportion of cans that were contaminated by the fire to within 0.02 with 90% confidence, we would need to randomly sample about 1692 cans from the warehouse.

8.39

a. Null hypothesis (H_0) : $H_0: \mu = 3.0$

Alternative hypothesis (H_a): $H_a: \mu > 3.0$

"μ" represents the population mean emotional empathy score for female college student.

b. $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Given that $\bar{x} = 3.28$, $\mu_0 = 3.0$, $\sigma = 0.5$, and $n = 30$

$$z = \frac{3.28 - 3.0}{\frac{0.5}{\sqrt{30}}} \approx 3.07$$

c. p-value is approximately 0.00107.

$1 - 0.99893 = 0.00107$.

d. The significance level, alpha, is the threshold below which the p-value is considered statistically significant, leading to the rejection of the null hypothesis.

Given that the p-value (0.00107) is less than $\alpha = 0.01$, we reject the null hypothesis.

This means that there is sufficient evidence to support the alternative hypothesis that the mean emotional empathy score for female college students is greater than 3.0.

e. The smallest significance level (α) that we can choose and still reject the null hypothesis is equal to the p-value of the test. This is because the p-value is the smallest level of significance at which the observed data would be considered statistically significant.

The smallest alpha value: 0.00107.