MA214 Lecture 3: Hypothesis Testing

3.00 Hypothesis Test.

- We want to validate a claim.
  - Test a hypothesis and see if they are true or false.

- Null hypothesis (H_0)
  - Provide a theory about the parameter value that we would like to reject.
- Alternative hypothesis (H_a).
  - Provide a theory that contradicts the null hypothesis.
  - A research hypothesis that we want to investigate.

But clearly not a fair fight!

Example:

Let p be the proportion of children in the population who prefer red truck.

Hypothesis to investigate: $p > 0.5$

$H_0: p = 0.5$, $H_a: p > 0.5$

Let X be the # of children in the sample who preferred the red truck.

A "reasonable" decision rule for concluding that the hypothesis is true is ...

Accept the research hypothesis if: Sample size: 18.

1. $X > 9$ 50%.
2. $X > 12$
3. $X > 15$
4. $X > 17$
5. Depends on the definition of "reasonable".

So:

if $p = 0.5$, i.e., the children have no preference.

then $P(X>9) = 0.4072$

Binomial distribution.

$n = 18$, $p = \frac{1}{2}$

n p

$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$ Compute each individual one and add them up!

$x = 0, 1, 2, 3, \ldots, n$

Thus there is 40% chance of incorrectly concluding that the children have a preference for red trucks!

True state of Nature.

Decision	H_0 is True	H_a is True
Accept H_0	Correct De. Prob = $1-\alpha$	Type II Error Prob. = β
Reject H_0	Type I Error Prob. = α	Correct De. Prob. = $1-\beta$

Power of a Test ($1-\beta$)

- The probability of correctly rejecting the null hypothesis.
  i.e. the probability that you accept the H_a.

The rejection rule:

Reject H_0 if $X > k$. // Accepts H_0 if $X \leq k$.

For sue values k: 9. 12. 15. 17.

β + Power = 1

- We want a larger Power.
  - With α and β being minimised.

BALANCE OUT!

Level of significance.

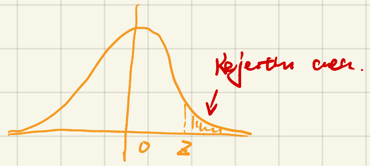
Sueros 5% or 1%.

Approaches to Hypothesis Testing.

- Critical value
- p-value

Large sample test of hypothesis about population mean.

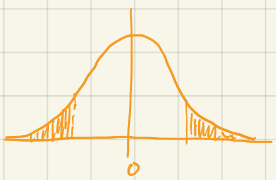
1. Write down H_0 and H_a. — σ is known. — σ is unknown.
2. Calculate the test statistic. — $Z = \frac{\bar{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$ — $Z = \frac{\bar{X}-\mu_0}{\frac{s}{\sqrt{n}}}$



H_0 $\mu = \mu_0$

H_a $\mu > \mu_0$

H_a $\mu < \mu_0$



H_a $\mu \neq \mu_0$

Example:

$H_0: \mu = 10$. $H_a: \mu < 10$

5% level of significance.

Test statistic:

$$Z = \frac{8.54 - 10}{\frac{3.25}{\sqrt{33}}} = -2.58$$

p-value: $P(Z < -2.58)$

Decision rule:

Reject H_0 if $Z < -z_{0.05}$ $\iff$ Reject if $Z < -1.645$.

- Observed Significance Level (or p-value)
  - Probability of observing a value of the test statistic that is at least as supportive of the alternative hypothesis as the one observed from the data.
- Decision Rule Based on p-value.
  - In hypothesis testing problem, the decision rule

- A smaller p value is more supportive of the Alternative Hypothesis.

Small sample test of hypothesis about population mean.

Example:

Given:

- $H_0: \mu = 5$. $H_a: \mu \neq 5$

$$t = \frac{2.9 - 5}{\frac{3.31}{\sqrt{10}}} = -2.0 \qquad DoF = N-1 = 10-1 = 9$$

Decision Rule:

Reject H_0 if $t < t_{0.025}$ OR $t > t_{0.025}$

This is a two-sided test.